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Transmission of electrons in a new type of disordered system

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Abstract. The distribution of transmission coefficients and Thouless numbers of electrons through a disordered system of finite size is studied both numerically and theoretically. The disordered system has a long-range structural correlation obeying an inverse-power law (generated by a modified Bernoulli map). We have found that

(i) the Lyapounov exponents of the transmission coefficient and the Thouless number are positive definite in an infinite system and

(ii) in a case of strong structural correlation the distribution of Lyapounov exponents of the transmission coefficient of a finite system converges slowly with increasing system size, and it does not obey the central-limit theorem.

1. Introduction

The one-electron problem in a one-dimensional random potential has been studied extensively by means of a variety of numerical and theoretical methods (Mott and Twose 1961, Hori 1968, Erdös and Herndon 1981, Economou 1983). In particular, localization of the wave function in disordered systems has been the most important problem to be solved. After an enormous amount of discussion, it has been established that in one dimension (1D) the disordered system has a pure point energy spectrum and its eigenfunctions are exponentially localized in an infinite system (Golidsheid *et al* 1977, Molcanov 1981). As a result, the ensemble-averaged conductance and transmission coefficient of a large enough system decrease exponentially with respect to the system size.

However, it is important to mention that most of the random potentials used up to now in these studies are of short-range structural correlation (SRSC). The SRSC means that the correlation function of the potential decreases exponentially or faster than it with respect to the distance, i.e. the correlation length is finite. These disordered systems of SRSC are believed to belong to one universality class in 1D. This idea has been suggested by the scaling theory of localization (Abrahams *et al* 1979). The idea of one universality class in each dimension is supported also by a qualitative argument with a real-space renormalization technique. By means of the real-space renormalization (Lee 1979, Aoki 1980, Sarker and Domany 1981) the Hamiltonian of the system with SRSC is expected to be translated into the renormalized Hamiltonian which has a shorter correlation length than that of the original one. Accordingly, these potentials of SRSC are expected to approach an ideal random potential after a sequence of the renormalization has been applied to it, because the finite correlation length decreases and finally vanishes by the successive operation of an appropriate real-space renormalization.

There is a different class of random potentials, whose correlation functions decrease according to a power law and hence the correlation lengths are divergent. Because the power-law correlation is invariant under this kind of real-space renormalization, the random potential does not approach an 'ideal random potential' by the successive operation of the renormalization.

We thus are interested in whether there are some differences in the behaviours of the above two types of random system.

In the separate papers (Aizawa *et al* 1991, Goda *et al* 1991), we have studied a fundamental feature of the electronic state in a disordered system with long-range structural correlation (LRSC) generated by a modified Bernoulli (MB) map. In these studies, we have obtained the following results.

(1) The Lyapounov (L) exponent of the wavefunction (at the band centre) is a positive constant in an infinite system for almost every sample (for $1 \leqslant B < 2$). The parameter B controls the correlation strength of the potential with LRSC.

(2) The distribution of the L exponent of the wavefunction for a finite system exhibits a slowly convergent property with respect to the system size when the structural correlation of the system becomes strong (for $\frac{3}{2} \leqslant B < 2$).

Because the spectral property and the distribution of the L exponent of wavefunctions are two fundamentals specifying the system, properties (1) and (2) must affect various physical phenomena of the systems.

Some workers have raised the objection that the transmission coefficient and the absolute square of the wavefunction are essentially the same quantities. In fact we can easily prove that the absolute values of the L exponents of these two quantities coincide in an infinite system. However, when we consider the distributions of these quantities for a finite system, the relation between them is not simple. It is necessary to study carefully these quantities in the new class of disordered systems, because an unusual feature has been observed in the distribution of the L exponents of the wavefunction for a finite system size. Therefore, the main purpose of the present paper is to study the distribution of two more realistic physical quantities: the transmission coefficient and the Thouless number in the random system with LRSC.

This paper is constructed as follows. In section 2 we give a brief review of the MB map and MB electronic system. In section 3 the transmission coefficient and Thouless number are numerically studied and analysed by adopting some scaling properties for them. Section 4 is devoted to the theoretical background which justifies the scaling properties adopted in section 3. Finally, the results obtained in this paper are summarized in section 5.

2. Preliminaries on the modified Bernoulli map

Consider a one-electron system described by a tight-binding Hamiltonian of the form

$$H = \sum_{n=0}^{N-1} |n\rangle \varepsilon_n \langle n| - \sum_{n=0}^{N-1} (|n\rangle\langle n+1| + |n+1\rangle\langle n|) \tag{2.1}$$

where the functions $\{|n\rangle\}$ denote an orthonormalized set of bases and the set of site energies $\{\varepsilon_n\}$ has been generated by a MB map.

The MB map is a one-dimensional map proposed in order to reveal the statistical properties of an intermittent chaos (Aizawa and Kohyama 1984):

$$X_{n+1}=f(X_n)=\begin{cases}X_n+2^{B-1}X_n^B & 0\leqslant X_n\leqslant \frac{1}{2}\\ X_n-2^{B-1}(1-X_n)^B & \frac{1}{2}\leqslant X_n\leqslant 1\end{cases} \tag{2.2}$$

where B is a non-negative bifurcation parameter which controls the strength of the structural correlation in the sequence $\{X_n\}$. This sequence $\{X_n\}$ is symbolized by the following rule:

$$\begin{aligned}&0\leqslant X_n\leqslant \tfrac{1}{2}\rightarrow \varepsilon_n=-\varepsilon\\ &\tfrac{1}{2}\leqslant X_n\leqslant 1\rightarrow \varepsilon_n=\varepsilon.\end{aligned} \tag{2.3}$$

We briefly review in the following some essential points of the symbolic sequence $\{\varepsilon_n\}$ characterized by the bifurcation parameter B on the bases of the results derived by Aizawa and Kohyama (1984), Aizawa (1984) and Aizawa *et al* (1984). Although the sequence $\{\varepsilon_n\}$ is created by the one-dimensional deterministic map, the correlation function decays exponentially or by a power law, because it is a chaotic sequence. So, we can regard the pseudo-random sequence $\{\varepsilon_n\}$ as a disordered sequence.

When B reaches the value 2 from below, the sequence $\{\varepsilon_n\}$ changes its characteristic drastically from stationary chaos characterized by the $\omega^{-\nu}(0\leqslant\nu<1)$ power spectrum for $\omega\ll 1$ to non-stationary chaos characterized by the $\omega^{-\nu}(\nu\geqslant 1)$ power spectrum. As a result, for $B\geqslant 2$ the sequence $\{\varepsilon_n\}$ of finite size becomes one of two kinds of pure sequence (i.e. $\{\varepsilon, \varepsilon, \varepsilon, \ldots, \varepsilon\}$ or $\{-\varepsilon, -\varepsilon, -\varepsilon, \ldots, -\varepsilon\}$) for almost every sample. In this paper we deal with only the stationary regime ($1\leqslant B<2$). The sequence $\{\varepsilon_n\}$ in this regime is rewritten in general as $\{(m_0, \sigma), (m_1, -\sigma), (m_2, \sigma), \ldots, (m_k, \sigma), (m_{k+1}, -\sigma), \ldots\}$. Here (m_k, σ) denotes the m_k times iteration of the same symbol σ, where σ represents ε or $-\varepsilon$. The distribution $P(m, \sigma)$ of the number m of times of the iteration in the pure sequence (m, σ) (called the residence time m in the paper of Aizawa *et al*) is

$$P(m,\sigma)=c[1+(B-1)m]^{-\beta} \tag{2.4}$$

which is independent of the value of the symbol σ, where $\beta=B/(B-1)$ and c is a normalization constant. The ensemble-averaged residence time $\langle m\rangle$ over the distribution (2.4) is finite for the stationary regime $1\leqslant B<2$ but is divergent for the non-stationary regime $B\geqslant 2$. Furthermore, the correlation function and the power spectrum of the sequence $\{\varepsilon_n\}$ are given as follows:

$$C(n)=\langle\varepsilon_0\varepsilon_n\rangle\varepsilon_0^2\sim\{1+(B-1)n\}^{(B-2)/(B-1)} \tag{2.5}$$

$$S(\omega)\sim\begin{cases}\omega^0 & 1<B<\frac{3}{2}\quad \omega\ll 1 & (2.6a)\\ \omega^{\beta-3} & \frac{3}{2}<B<2\quad \omega\ll 1. & (2.6b)\end{cases}$$

Confirming $-\infty<(B-2)/(B-1)<0$ for $1<B<2$, equation (2.5) describes an inverse-power-law structural correlation for $m\gg 1$ in the sequence $\{\varepsilon_n\}$. In the range of the bifurcation parameter $1<B<\frac{3}{2}$, the white power spectrum in equation (2.6) shows that the corresponding sequence $\{\varepsilon_n\}$ has only SRSC. In the range $\frac{3}{2}<B<2$ the second moment $\langle m^2\rangle$ of the residence time diverges and this is the case on which we shall focus our attention in this paper.

Now we return to the Hamiltonian (2.1). When the symbolic sequence $\{\varepsilon_n\}$ represents the sequence of site energies as in the Hamiltonian (2.1), we call the residence time m the cluster size m. The Schrödinger equation described by it is $\varphi_{n+1}=(E-\varepsilon_n)\varphi_n-\varphi_{n-1}$, where φ_n is the amplitude of the wavefunction at site n. The equation is rewritten in the form

$$\begin{pmatrix}\varphi_{n+1}\\ \varphi_n\end{pmatrix} = \mathbf{M}_n \begin{pmatrix}\varphi_1\\ \varphi_0\end{pmatrix} = \prod_{i=1}^{n} \mathbf{T}_i \begin{pmatrix}\varphi_1\\ \varphi_0\end{pmatrix} \tag{2.7}$$

$$\mathbf{T}_i = \begin{pmatrix}E-\varepsilon_i & -1\\ 1 & 0\end{pmatrix} \tag{2.8}$$

where $\mathbf{T}_i$ is the transfer matrix.

3. Numerical results: transmission coefficient and Thouless number

The transmission coefficient $T(N)$ of a finite system with system size N, of band centre energy ($E = 0$) as a typical example, is given as

$$T(N) = 4(\|\mathbf{M}_N\|^2 + 2)^{-1} \tag{3.1}$$

where $\|\mathbf{M}_N\|^2$ denotes the sum of the squares of each element of the transfer matrix $\mathbf{M}_N$ in equation (2.7) (Stone *et al* 1981). The transmission coefficient depends only on the transfer matrix $\mathbf{M}_N$ itself and is independent of the boundary condition, differing from some other physical quantities depending on the wavefunction.

The Thouless number $g(N)$, which reflects the property of eigenfunctions and describes the conductance of the system, is defined as

$$g(N) = \Delta E_\alpha(N)/\Delta W(N) \tag{3.2}$$

where $\Delta E_\alpha(N)$ is the shift of the αth energy level due to changing the boundary condition from a periodic one to an antiperiodic one, and $\Delta W(N)$ is the mean spacing of its energy levels (Thouless 1974). The Thouless number has been used to distinguish whether the eigenstate is localized or not. We select in this paper, as a typical example, the middle member of the eigenstates with respect to the energy as the αth state, by considering only the system with an odd system size N.

Figures 1(*a*) and 1(*b*) show two sample averages, $\langle \ln T(N)\rangle$ and $\langle \ln g(N)\rangle$, for some values of the bifurcation parameter B, where $\langle \ldots \rangle$ denotes a sample average. The 2^{15} samples of the same size N are selected from a huge sequence $\{\varepsilon_n\}$ starting from an initial value X_0 in the mapping (2.2). It seems in figure 1(*a*) and 1(*b*) as if $\langle \ln T(N)\rangle$ and $\langle \ln g(N)\rangle$ decrease exponentially with increasing system size N.

Hence we calculate numerically the distribution of the L exponents of the transmission coefficient $-[\ln T(N)]/N$ ($=\gamma$) which is of finite size and investigate its asymptotic behaviour as it approaches an infinite system, in order to understand the transmission coefficient in more detail.

Figures 2(*a*) and 2(*b*) show the histograms of the distribution of γ over 2^{17} samples. (The potential strength ε has been taken to have the value 0.6.) Just as has been observed in the L exponent of wavefunctions in the paper by Goda *et al* (1991), two kinds of distribution coexist also in the distribution of the L exponent γ of the transmission coefficient $T(N)$ for the case $B = 1.7$. This double-peak structure of the distribution comes from the LRSC of the MB map.

As has been mentioned in the introduction, it is not obvious that the distribution of the transmission coefficients is the same as that of the wavefunctions in the finite-size

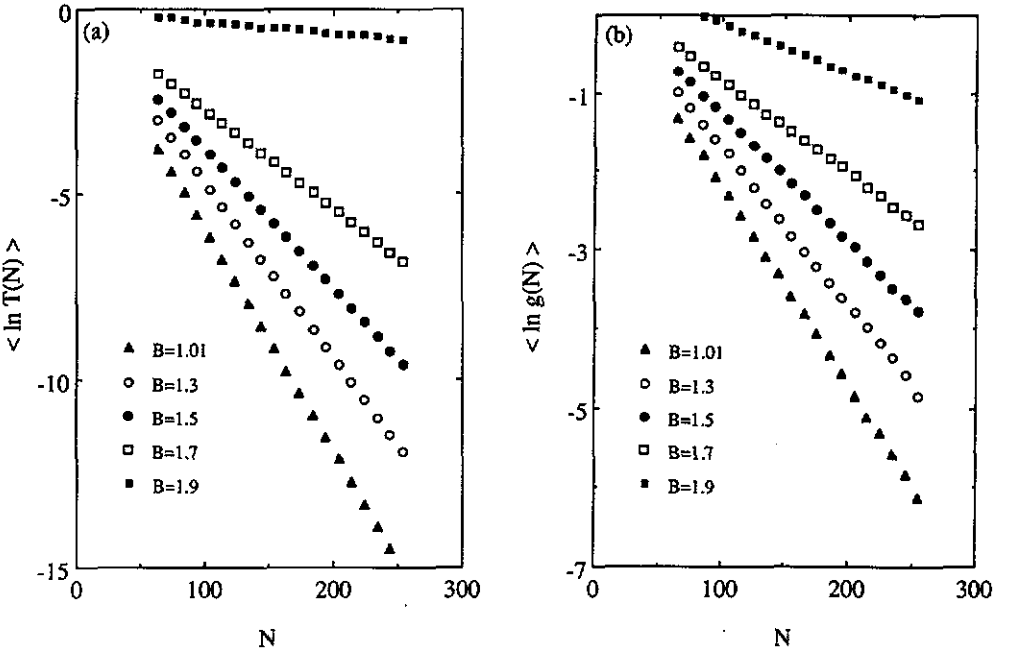

Figure 1. The numerically determined average of the (a) transmission coefficient $\langle \ln T(N) \rangle$ and (b) Thouless number $\langle \ln g(N) \rangle$ as functions of the system size N. The potential strength ε has been given the value 0.5.

system. What kind of property is derived from the distribution of the transmission coefficients? To understand the details of the distribution we define the operation $\langle \dots \rangle_2$ as the average over parts in the 2^{17} samples (forming the second distribution). This subset of samples is obtained by subtracting the samples composed of pure clusters (forming a sharp distribution around $\gamma = 0$ in the distribution) from the whole sample as in the paper by Goda *et al.* For a fixed N, the number of samples concerned with the average $\langle \dots \rangle_2$ decreases more and more, when the value of B increases and becomes close to 2.

For example, the N-dependence of $\langle \gamma \rangle$ and $\langle \gamma \rangle_2$ for $B = 1.7$ and $B = 1.9$ are shown in figures 3(a) and 3(b). To infer the sampling uncertainty, the data from systems with different initial values X_0 of the mapping (2.2) are plotted in figures 3(a) and 3(b). By considering numerically the asymptotic behaviour of the L exponent γ with increasing system size N, we found the following scaling form:

$$\langle \gamma \rangle = \text{constant} \tag{3.3}$$

$$\langle \gamma \rangle_2 = c_2(B) N^{-\lambda(B)} + \gamma_\infty(B). \tag{3.4}$$

The scaling form of the L exponent will be confirmed by a theoretical argument in section 4. Figures 4 and 5 show numerical values of $\lambda(B)$ and $\gamma_\infty(B)$, respectively, determined by least-squares fit in equation (3.4). The value of $\lambda(B)$ approaches zero when B approaches the value 2.0. In other words, $\langle \gamma \rangle_2$ becomes independent of N when $B = 2.0$, where the stationary–non-stationary chaos transition occurs in the MB map. Concerning $\gamma_\infty(B)$, which describes the L exponent γ of the transmission coefficient in an infinite system, the value decreases with increasing bifurcation parameter B. Although the L exponent $\gamma_\infty(B)$ in an infinite system is expected to vanish at $B = 2.0$, the functional

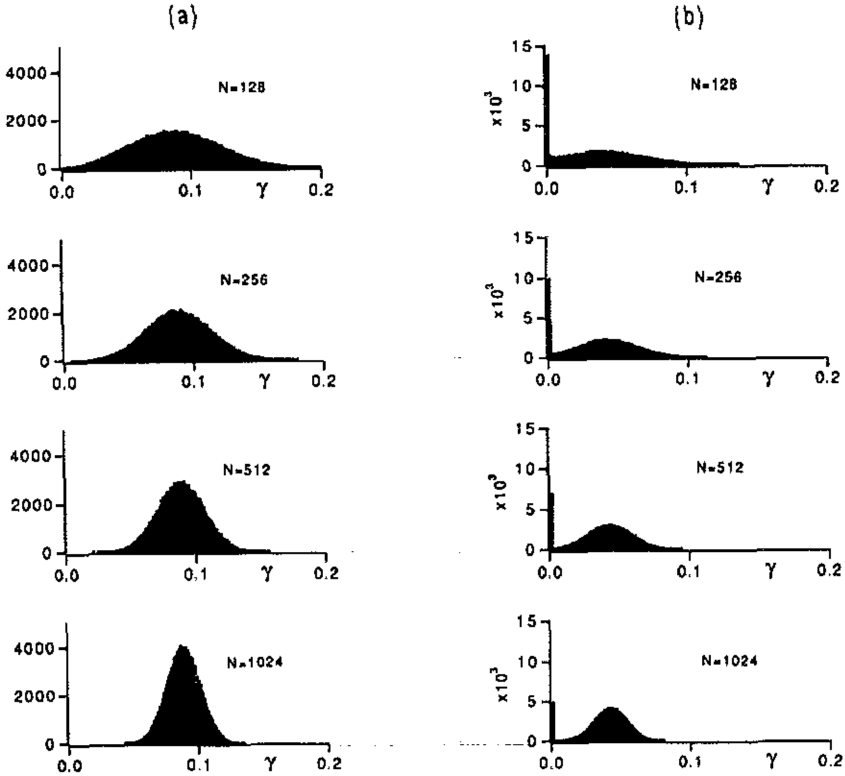

Figure 2. The histogram of the L exponent of the transmission coefficient of a finite system with system sizes $N = 128, 256, 512$ and 1024, obtained numerically from an ensemble of 2^{17} systems. The potential strength ε has been given the value 0.6 and the bifurcation parameters B are (*a*) 1.01 and (*b*) 1.7.

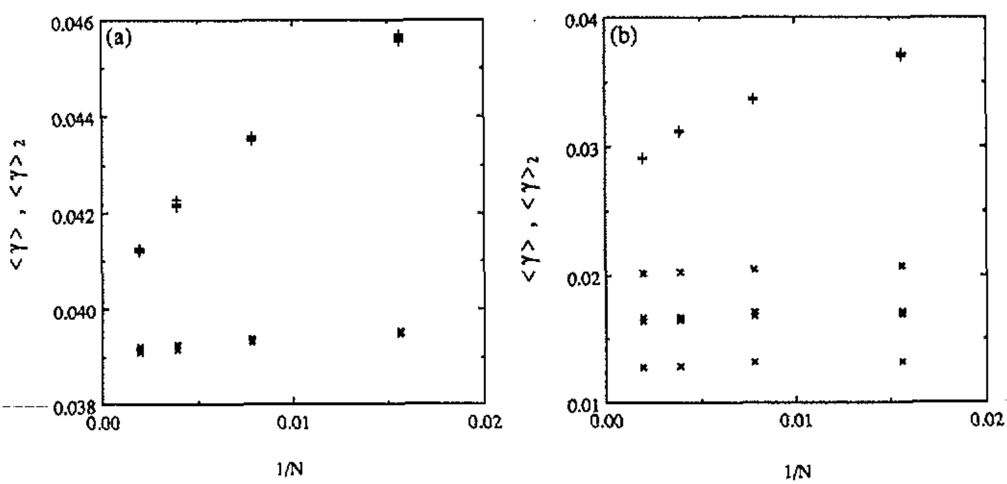

Figure 3. N-dependences of the L exponent $\langle\gamma\rangle$ of finite size ($\times$) and the exponent $\langle\gamma\rangle_2$ on the second distribution (+). The bifurcation parameters B are (*a*) 1.7 and (*b*) 1.9. The numerical data obtained for four differential initial values $X_0 = 1/\sqrt{2}, 1/\sqrt{3}, 3/5$ and $4/5$ are plotted.

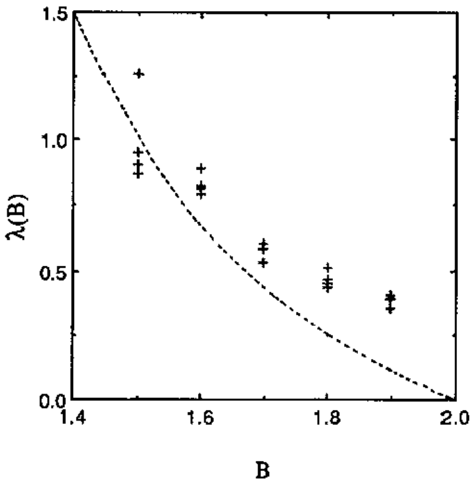

Figure 4. Estimated value of the exponent $\lambda(B)$ in equation (3.4) as a function of bifurcation parameter B: $+$, numerical data for $\lambda(B)$ obtained for four different initial values $X_0 = 1/\sqrt{2}, 1/\sqrt{3}, 3/5$ and $4/5$; ---, $\lambda(B) = (2 - B)/(B - 1)$ in equation (4.8) estimated theoretically.

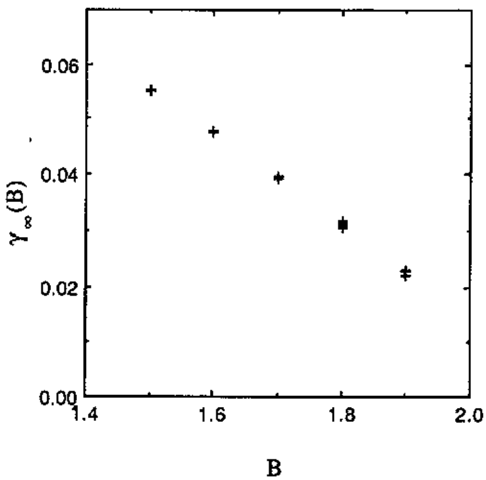

Figure 5. Estimated value of the L exponent γ_∞ (from equation (3.4)) as a function of the bifurcation parameter B. The numerical data for $\gamma_\infty(B)$ obtained for four different initial values $X_0 = 1/\sqrt{2}, 1/\sqrt{3}, 3/5$ and $4/5$ are plotted.

form of $\gamma_\infty(B)$ near $B = 2.0$ is very hard to estimate from our numerical data, because numerical accuracy of the distribution of γ is extremely incorrect for B close to the value 2. However, to compensate for the numerical insufficiency on this point, a discussion will be given in section 4 to clarify the nature of $\gamma_\infty(B)$ near $B = 2.0$.

Moreover, we have confirmed numerically that the L exponent of the Thouless number also has a positive value for $1 \leqslant B < 2$ in the same way as above.

Next, consider the fluctuations of γ from $\langle\gamma\rangle_2$ in the second distribution. The scaling form

$$\sqrt{\langle(\Delta\gamma)^2\rangle_2} \propto N^{-\chi(B)} \tag{3.5}$$

is found numerically by fitting the data, where $\Delta\gamma$ is $\gamma - \langle\gamma\rangle_2$. The estimated value of $\chi(B)$ is plotted in figure 6. For $1 \leqslant B < \frac{3}{2}$, the value of $\chi(B)$ is roughly $\frac{1}{2}$, implying that the convergent property of the distribution of γ with respect to N obeys or approximately obeys the central-limit theorem (CLT). However, it is surprising that for $\frac{3}{2} \leqslant B < 2$ the distribution converges more slowly than that obeying the CLT. In our numerical data used in figure 6, the local slope in the ln–ln plot of equation (3.5) slightly depends on the system size N. For $1 \leqslant B \leqslant \frac{3}{2}$, the local slope for a comparably large system size N (around $N \simeq 2^{10}$ in figure 6) tends to be closer to the value $\frac{1}{2}$ than the data in figure 6. For $\frac{8}{5} \leqslant B < 2$, it tends to decrease less than that in figure 6. Owing to the strong and long-range correlation between local structures of the sequence $\{\varepsilon_n\}$, the L exponents γ of subsystems in a huge sample still have a long-range correlation. This corresponds to the large deviation property of the symbolic sequence $\{\varepsilon_n\}$ (Aizawa 1989). All these properties of $T(N)$ agree with those of the wavefunctions.

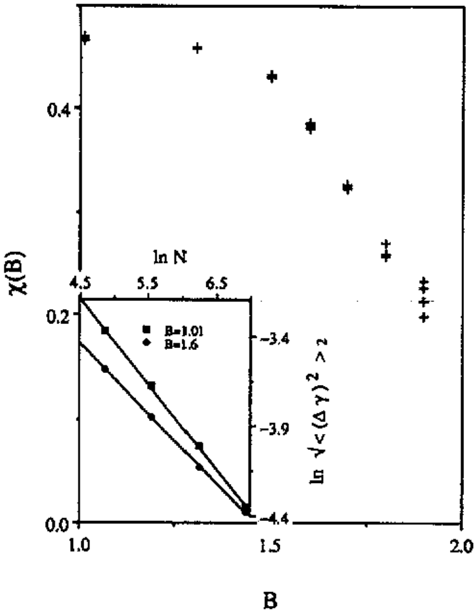

Figure 6. The exponent $\chi(B)$ of the power for the standard deviation of the L exponent of the transmission coefficient as a function of the bifurcation parameter B. The numerical data for $\chi(B)$ obtained for four different initial values $X_0 = 1/\sqrt{2}, 1/\sqrt{3}, 3/5$ and $4/5$ are plotted. The inset shows some data of the standard deviation as a function of system size: ——, least-squares fit.

Now, consider the beta function of the transmission coefficients (Abrahams *et al* 1979, Lee and Ramakrishnan 1985). From the assumed form (3.3) and (3.4), we can derive that

$$\beta(N) \equiv \partial\langle \ln T(N)\rangle/\partial(\ln N) = \langle \ln T(N)\rangle \tag{3.6}$$

$$\beta_2(N) \equiv \partial\langle \ln T(N)\rangle_2/\partial(\ln N) = \langle \ln T(N)\rangle_2 + c_2(B)\lambda(B)N^{1-\lambda(B)}. \tag{3.7}$$

That is,

$$\beta_2(N) = Y + AY^{1-\lambda}\{1 + DY^{-\lambda}[1 + DY^{-\lambda}(\ldots)^{1-\lambda}]^{1-\lambda}\}^{1-\lambda} \qquad (|Y| \gg 1) \tag{3.7'}$$

where $Y = \langle \ln T(N)\rangle_2$, $A = \lambda(B)c_2(B)[-\gamma_\infty(B)]^\lambda$, $D = c_2(B)[-\gamma_\infty(B)]^{\lambda-1}$. The above $\beta_2(N)$ has a correction term $c_2(B)\lambda(B)N^{1-\lambda(B)}$ in it, which is not in the scaling function of an ideal random system. The leading term of the correction term in $\beta_2(N)$ is $AY^{1-\lambda}$ for $|Y| \gg 1$ which depends on the value of B. Figures 7(*a*) and 7(*b*) show the numerical results of $\beta(N)$ and $\beta_2(N)$ calculated from the data in figure 1(*a*). Numerically we cannot see a clear difference between the two scaling functions $\beta(N)$ and $\beta_2(N)$ in the figures because the correction term in equation (3.7) is small. Figures 8(*a*) and 8(*b*) show the beta functions of the Thouless number from the data in figure 1(*b*).

4. Discussion based on Furstenberg's theorem

In this section we try to apply the Furstenberg (F) (1963) convergent theorem on a product of random matrices to a random system with LRSC obeying the inverse-power law.

An essential point on applying it is that the sequence $\{\varepsilon_n\}$ which we deal with is a purely random sequence of pure clusters (called a renewal process) with a pure cluster

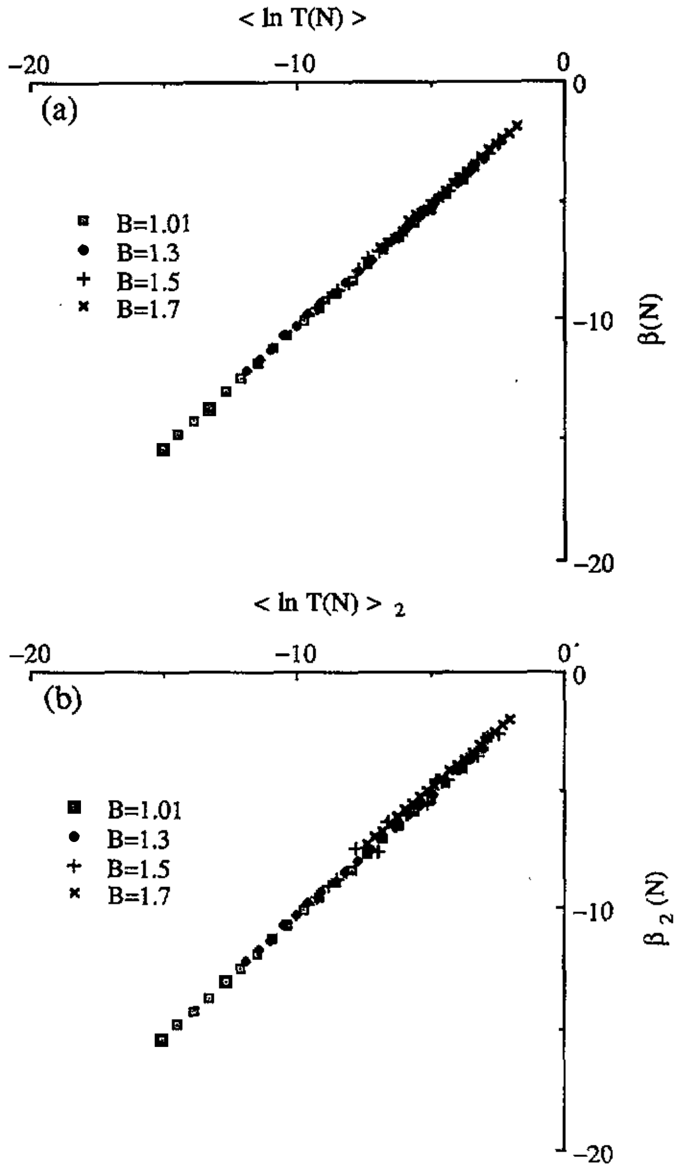

Figure 7. (*a*) The beta function β of the transmission coefficient numerically calculated from the data in figure 1(*a*). (*b*) The beta function β_2 obtained for average operation $\langle\ldots\rangle_2$.

size distribution $P(m) \sim m^{-\beta}$. Then the m iterations of symbol σ and the adjacent n iterations of symbol $-\sigma$ have a joint probability

$$P(m, n) \sim m^{-\beta} n^{-\beta}. \tag{4.1}$$

The transfer matrix corresponding to this event is

$$\mathbf{X}(m, n) = \begin{pmatrix} -\sigma & -1 \\ 1 & 0 \end{pmatrix}^n \begin{pmatrix} \sigma & -1 \\ 1 & 0 \end{pmatrix}^m. \tag{4.2}$$

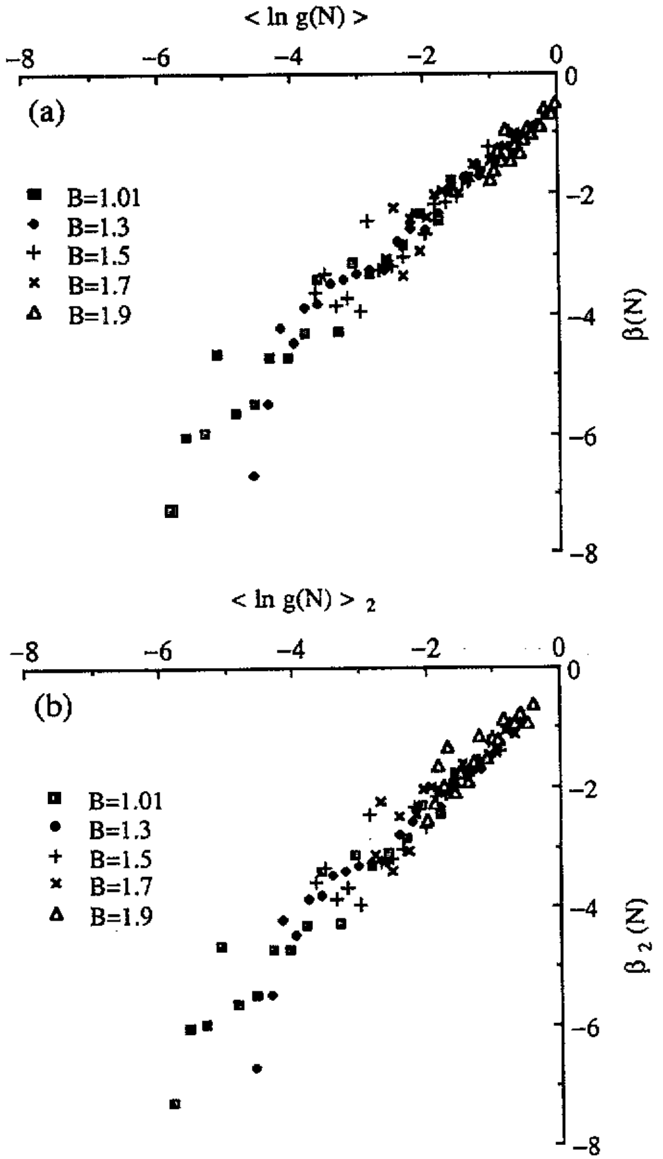

Figure 8. (*a*) The beta function β of the Thouless number numerically calculated from the data in figure 1(*b*). (*b*) The beta function β_2 obtained for average operation $\langle\ldots\rangle_2$.

The sequence $\{\mathbf{X}(m_i, n_i)\}$ $(i = 0\text{–}\infty)$ is composed of mutually independent stochastic variables $\mathbf{X}(m_i, n_i)$ in the special linear group $\mathrm{SL}(2, R)$ with a common distribution $P(m,n)$. Let G denote the smallest closed subgroup of $\mathrm{SL}(2, R)$ containing the support of the common distribution $P(m, n)$. Then if G contains at least two elements of $\mathrm{SL}(2, R)$ with common eigenvectors, under an additional condition

$$\int_{\mathrm{SL}(2,R)} |||\mathbf{X}(m,n)|||\, \mathrm{d}\mu(\mathbf{X}(m,n)) < \infty \tag{4.3}$$

where $|||\mathbf{X}(m,n)||| = \sup_{\|v_0\|=1}\|\mathbf{X}(m,n)v_0\|$ it is a sufficient condition of the F condition for establishing the F theorem (Matsuda and Ishii 1970, Ishii 1973).

Here, we consider for example two elements of SL(2, R): $\mathbf{X}(1,2)$, $\mathbf{X}(2,1)$. Then, there are no common eigenvectors in these two matrices, and the additional condition (4.3) is also satisfied for the common distribution (4.1) for $B<2$. It is thus proved that the F condition is satisfied for $E=0$. Moreover, it is proved that, at least, the F condition is satisfied for any energy except for several special energies. Accordingly, we get from the F theorem

$$\mathrm{Prob}\left\{\lim_{L\to\infty}\left[\frac{1}{L}\ln\left(\left\|\prod_{j=0}^{L}\mathbf{X}(m_j,n_j)u_0\right\|\right)\right]=Q(B)>0\right\}=1 \qquad 1\leqslant B<2 \tag{4.4}$$

for $u_0=(\varphi_1,\varphi_0)^{\mathrm{T}}\in R^2-\{0\}$ for $E=0$.

As for the real system size N, it is reasonable to expect $N\simeq 2L\langle m\rangle$ for a sufficiently large system, where $\langle m\rangle$ is the mean cluster size. Then we have

$$\lim_{N\to\infty}\left[\frac{1}{N}\left(\left\|\prod_{k}^{N}\mathbf{M}_k u_0\right\|\right)\right]=\frac{1}{2\langle m\rangle}\lim_{L\to\infty}\left[\frac{1}{L}\ln\left(\left\|\prod_{j}^{L}\mathbf{X}(m_j,n_j)u_0\right\|\right)\right]=\frac{Q(B)}{2\langle m\rangle}. \tag{4.5}$$

If the mean cluster size $\langle m\rangle$ is finite $(1\leqslant B<2)$, the quantity $(\varphi_{N+1}^2+\varphi_N^2)$ of the wavefunction in the random systems increases exponentially with increasing system size with probability 1 for any initial vector $u_0\in R^2-\{0\}$. In addition it has been known that, for large N, the L exponent of the wavefunctions is equal to the corresponding exponent of the transmission coefficients. Accordingly, $\langle\gamma\rangle$ in section 3 is a positive constant for $N\gg\langle m\rangle$, i.e., $T(N)$ decreases exponentially with increasing system size N with probability 1. These results based on the F theorem are consistent with Kotani's (1984) theory which states that one-dimensional Schrödinger operators with an ergodic and stationary random potential have a positive L exponent of wavefunction with probability 1.

Now the N-dependence of $\langle\gamma\rangle_2$ is estimated by use of the probability $R(N)$ that two pure clusters appear with size N, defined by

$$R(N)=\sum_{m=N}^{\infty}P(m)\frac{(m-N+1)}{N}\Big/\sum_{m=1}^{\infty}P(m)\frac{m}{N}\sim N^{(B-2)/(B-1)}. \tag{4.6}$$

Accordingly we have

$$\langle\gamma\rangle=R(N)\Gamma(N)-[1-R(N)]\langle\gamma\rangle_2 \tag{4.7}$$

where $\Gamma(N)$ corresponds to a γ of periodic sequence with size N and is of the order of N^{-1}. From equation (4.6), $\langle\gamma\rangle_2$ is estimated approximately as

$$\langle\gamma\rangle_2\simeq\langle\gamma\rangle(1+cN^{(B-2)/(B-1)}) \tag{4.8}$$

where c is the normalization constant in equation (4.6). Equation (4.7) supports the inverse-power law equation (3.4) for the N-dependence of $\langle\gamma\rangle_2$ numerically found in section 3. The value of $\lambda(B)$ in figure 3(a) for $B<2$ is, however, different from the exponent $-(B-2)/(B-1)$ in equation (4.7), although the overall features of the B-dependence coincide. This disagreement arises because of the finite size effects or shortage of sample and these influences are increasingly enhanced as B becomes close to 2.

Finally we mention the L exponent $\gamma_\infty(B)$ near $B = 2$. In the expression for the L exponent in equation (4.4) we have shown that the mean cluster size $\langle m \rangle$ diverges for $B = 2$ according to equation (4.5). On the other hand the quantity $Q(B)$ does not seem to change drastically near $B = 2$. It seems to be finite. The L exponent $\gamma_\infty(B)$ in an infinite system is thus expected to decrease continuously when B approaches the value 2 and finally to vanish at $B = 2$.

5. Summary and conclusions

We have studied the L exponent of the transmission coefficient γ in a disordered system with LRSC generated by the MB map by means of numerical and theoretical methods.

The results obtained in this paper are summarized as follows.

(1) The mean value of the L exponent $\langle \gamma \rangle$ of the transmission coefficient of a finite system is a positive constant with respect to a system size N for $N \gg \langle m \rangle$ for $1 \leqslant B < 2$. This agrees with the results of ordinary random systems.

(2) The L exponent $\langle \gamma \rangle$ in an infinite system is thus positive for $1 \leqslant B < 2$ but is expected to decrease continuously when B approaches 2 and finally to vanish at $B = 2$.

(3) $\langle \gamma \rangle_2$ has a scaling form for $N \gg \langle m \rangle$ given in equation (3.4) with respect to the system size N. The beta function $\beta_2(N)$ of the transmission coefficient is different from that of ordinary random systems.

(4) The convergent properties of the distribution of transmission coefficient with respect to the system size N do not obey the CLT at least for $\frac{3}{2} < B < 2$. The slow convergence corresponds to the large deviation property of the symbolic sequence $\{\varepsilon_n\}$.

(5) Statement (1) has been confirmed also for the Thouless number.

It should be noted that the results obtained in this paper are well analysed by the use of a renewal process which has been found in the sequence $\{\varepsilon_n\}$. When the system with a continuous version of the potential as $\{X_n\}$ is constructed, we cannot utilize a powerful tool such as the renewal process. It is thus not clear at this stage whether the continuous version of the MB electronic system also exhibits the same results. This problem remains for future study.

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